Theoretical economics study of the formation of Defense alliances

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THEORETICAL ECONOMICS STUDY OF THE FORMATION OF DEFENSE ALLIANCES

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Primarily, I created an alliance model with and without risk perceptions based on the joint product model and the self-protection model. Next, I examined the behavior of countries in the model. Finally, I considered the implications of the results. The characters of the countries may influence the alliance burden sharing and productivity of defense expenditure whose effects on the country's utilities influences the alliance formation.

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THEORETICAL ECONOMICS STUDY OF THE FORMATION OF DEFENSE ALLIANCES

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ABSTRACT

This thesis is a theoretical economics study of the formation of defense alliances. It discusses the probability that countries, which are characterized by both their perceptions about benefits and risks of alliance membership and the gross domestic product, form an alliance.

Primarily, I created an alliance model with and without risk perceptions based on the joint product model and the self-protection model. Next, I examined the behavior of countries in the model. Finally, I considered the implications of the results. The characteristics of the countries may influence the alliance burden sharing and productivity of defense expenditure whose effects on the country's utilities influences the alliance formation.

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I. INTRODUCTION

A. BACKGROUND

The study of military alliances using economic theory has continued since Olson and Zeckhauser (1966) first applied the public goods theory to this subject. [Ref. 1] Their study and others following have focused mainly on the North Atlantic Treaty Organization (NATO). NATO observed that the Warsaw Treaty Organization (WTO) was a common enemy in a bipolar world system. NATO was therefore designed to protect its members from WTO aggression. In fact, the NATO alliance has deterred WTO aggression against its members and its members have correspondingly benefited from the alliance. Most of these previous studies view the alliance from a positive standpoint, except for some problems, such as disproportionate burden sharing. [Ref. 2]

In other regions, such as East Asia, military alliances like NATO do not exist. However, some two-country-alliances do exist, such as the United States-Japan Treaty. From the standpoint of economic theory, little is known about such alliances. The common enemy is much less obvious than it was for Western Europe during the Cold War. Thus, joining an alliance in this circumstance would not always benefit all allies. For example, an ally could be involved in a war in which they would not have been involved had it not been for the alliance. Some countries consider the main benefit of an alliance is deterrence while some countries think alliances are risky.

The question addressed here is how the perceptions of member countries influence the formation of an alliance. Depending on perceptions, do they join an alliance or remain independent?

This paper is a theoretical economic study on the formation of defense alliances. It will analyze perceptions of countries in an alliance and discuss the

effects of these perceptions on alliance formation. Finally, it will discuss the implications for alliance formation.

B. OBJECTIVES

This paper discusses the probability that countries, which are characterized by both their perceptions about the benefits and risks of alliance membership and their gross domestic product, form an alliance. It also discusses the prospective effect of changes in the size of the countries on alliance formation.

C. SCOPE AND LIMITATION

This paper considers the formation of a defense alliance between two countries. Although I acknowledge that there are uncountable factors that influence the formation of alliances and relationships between countries, such as economic, ideological, racial, religious and historical factors, I would like to focus on the country's perception of the risk and benefit from an alliance.

The countries are assumed to be almost identical and are democratic and capitalist. They relate to each other so much economically that a war against one of them can influence the other country. For this reason, they do not prefer war and think of their defense power as a deterrence. Besides, their utility is measured by the amount of money they can spend for private goods. In addition, this paper will discuss the formation of alliances based on two assumptions.

1. Assumption 1

The countries discussed are assumed to be rational players. They are not assumed to take reckless actions. They make decisions by themselves on whether or not to join an alliance and seek to maximize their utility.

2. Assumption 2

Defense expenditures are determined by a Nash equilibrium. The country decides on the defense expenditure that maximizes its utility after observing the defense expenditures selected by the other ally. The second country reacts in

the same way. As these actions continue, their defense expenditures approach a Nash equilibrium.

D. METHODOLOGY

- Research previous studies on alliances using economic theory.
- Create an alliance model without risk perception based on the joint product model and the self-protection.
- Develop an alliance model with risk from the alliance model without risk.
- Examine the behavior of countries using the alliance model with risk.
- Consider the implications of the model results.

E. LITERATURE REVIEW

There have been several models employed to examine economic theory in military alliances. One model is a purely public model introduced by Olson and Zeckhauser. It focuses mainly on the North Atlantic Treaty Organization (NATO) and discusses an alliance as a pure public good. Another model is the joint product model employed by Sandler. The model was developed from the purely public model and discusses an alliance as an impurely public model. [Refs. 2 and 3] The other type of model is the self-protection model constructed by Ehrich and Becker.

1. Purely Public Product Model

Olson and Zeckhauser applied economic theory to the study of military alliances initially in 1966. [Ref. 1] They discussed NATO and observed that WTO was a common enemy of NATO and that deterring WTO aggression against any NATO members was in the common interest of all members. In the 1950s and 1960s, NATO depended primarily on deterrence and mainly on the U.S. stockpile of strategic nuclear weapons. Reliance on nuclear weapons was essential since the conventional forces of NATO were much smaller than that of the Russian forces. From these observations, a military alliance is defined as a

group of countries that provides protection to all members from aggression by common enemies. Accordingly, Olson and Zeckhauser considered deterrence as a pure public good. [Ref. 1] Measuring the size of an ally by its Gross Domestic Product (GDP), they examined the effect of GDP on defense expenditures as a percent of GDP in a Nash equilibrium. Olson and Zeckhauser also assumed that all allies are equal to one another in every aspect other than the GDP. [Ref. 1] In particular, the allies share the same perception of the threat and the effect of an alliance on deterrence. Under these assumptions, Olson and Zechauser explain the formation of NATO predicting disproportionality in defense burden sharing; larger allies shoulder the defense burden of the smaller allies. [Ref. 1] In their studies, despite the problems of disproportionality, countries are always better off cooperating than being isolated.

2. The Joint Product Model

Extending the pure public goods theory, Sandler employed a joint product model to explain observed reductions in disproportionality (as measured by defense expenditures as a percent of GDP) between the rich and poor allies in NATO during the 1970s and 1980s. [Ref. 4] In the joint product model, military expenditures are viewed as being purely public, impurely public, and private goods. For example, purely public goods include deterrence provided by strategic nuclear weapons, impurely public goods include damage limiting protection provided by conventional and tactical nuclear weapons, and private goods include forces policing terrorist activity at home. A country like the United States maintains both nuclear and conventional forces and uses them for a variety of both public and private purposes. In addition, NATO placed increasing reliance on conventional weapons relative to nuclear weapons under the doctrine of flexible response in the 1970s. From these observations, Sandler and others explain the formation of NATO by using the joint product model.

The joint product model appears to add more understanding to the NATO alliance. In particular, the model predicts that technological and strategic changes, associated with an alliance arsenal, can alter the mix of public benefits. However, the mix of public and private benefits is uniquely determined by the mix of weapons in the alliance's arsenal and the alliance's defense strategy, e.g., flexible response. This model also concludes that allies are never at a disadvantage from the alliance.

3. The Self-Protection Model

From another point of view, Ehrlich and Becker (1972) discuss a defense model as the 'self-protection' Model. [Ref. 5] With this model, they studied the behavior of a country that is assumed to seek its maximum expected utility; maximum expected utility combines utilities at peacetime and at during war, using the probability of war. Toshitaka Fukiharu applied the self-protection model to explain alliances and arms races between two countries. [Ref. 6]

I consider it inappropriate to maintain an alliance that benefits all potential allies. Other studies have assumed that an alliance benefits all allies and have no doubt that it is better for a country to cooperate than to be isolated. This assumption comes from the circumstances surrounding NATO during the Cold War. It is inappropriate in other regions, and especially after the collapse of the bipolar system of NATO and WTO. Countries do not share a perception of a common enemy in the other regions. In other words, countries observe their unique enemies regarding their individual national interests. In international relations, it is good for a country that an alliance would protect it and would deter war against its adversary; however, there is a risk that the alliance would force a country to contribute to a war initiated against another ally. There is also a risk that the alliance would destabilize the balance of power and increase the probability of war. Some countries may perceive an alliance as a benefit; other countries may not.

Taking international relations into account, this paper will discuss how the perceptions of countries influence the formation of an alliance. Does a country cooperate, or does a country become isolated? Examining behaviors of countries with mathematical models, this paper will consider the implications of these perceptions on alliance formation.

II. MODEL DESCRIPTION

A. ALLIANCE MODEL WITHOUT RISK PERCEPTION

Consider an original alliance model without risk perception based on the previous studies as the first step in analyzing the behavior of the countries regarding an alliance. In the model, defense goods are considered as purely or impurely public goods, the utilities of a country are measured by the expenditures for private goods, and expected utility depends on both the probabilities of peace and war and the utilities at peace and at war. The objective of a country is to maximize its expected utility subject to its resource constraint of the Gross Domestic Product (GDP).

1. Model Structure

Consider the alliance model without risk perception where there are two friendly countries, Country 1 and Country 2, and their adversaries, Enemy A and Enemy B. As both countries are rational players, each country independently decides on its defense expenditure by maximizing its utility subject to its income constraint.

In addition, there is the possibility of two wars. One war is War A where Enemy A would attack Country 1, the other war is War B where Enemy B would attack Country 2. Countries 1 and 2 suffer losses from both War A and War B. For these reasons, Country 1 and Country 2 are interested in a bilateral military alliance to prevent both wars.

In the model, let the notation be as follows:

i and j	=	Friendly countries: 1 or 2
k and m	=	Enemies and wars: A or B *
s	=	Situation: 0, 1, 2, 3 *
U_i	=	Expected utility of Country i
U_i^s	=	Utility of Country i under Situation s
G_i	=	Gross domestic product of Country i
X_i	=	The quantity of private goods purchased by Country i
Y_i	=	The quantity of defense goods purchased by Country i
L_i^s	=	Loss that Country i suffers under Situation s
E_{ij}	=	Portion of Country j's defense goods which contribute to Country i
Z_i	=	Country i's deterrence
P_i^s	=	Probability that Situation s occurs
a_i	=	Utility coefficient of Country i
p_i		Price of private goods in Country i
l_{is}	=	Coefficient for Country i's loss suffered from situations that Country i estimates
d_i^k	=	Effectiveness coefficient of deterrence of War k that Country i estimates
q_i^k	=	Probability of War k that Country i estimates

*

- **Wars are defined as follows:**

  War A: War where Enemy A attacks Country 1
  War B: War where Enemy B attacks Country 2

- **Situations are defined as follows:**

  Situation 0: Situation where neither War A nor War B occurs
  Situation 1: Situation where War A occurs
  Situation 2: Situation where War B occurs
  Situation 3: Situation where both War A and War B occur

Under the alliance, Z_i represents Country i's total consumption of the defense goods or its deterrence.

$$Z_i = Y_i + E_{ij}Y_j \quad (0 \le E_{ij} \le 1)$$

Though Country i produces Y_i of defense expenditure, it perceives that E_{ij} of Y_j is relevant to Country j's defense. In other words, Country i perceives that Country j contributes E_{ij} of its defense expenditure to Country i. In general, E_{ij} should be a value between 0 and 1. If $E_{ij} = 1$, Country j's defense expenditures are purely public. Conversely, if $E_{ij} = 0$, its defense expenditures are purely private. That is, Country i perceives no commitment to its defense from Country j and Country i cannot depend on Country j's assistance in its defense calculations. [Ref. 7]

2. Objective

The objective of the country in this model is to maximize its expected utility composed of the probabilities of peace and wars and its utilities at peace and at war.

3. Probabilities of Peace and Wars

Assume that the probability of War k, or q_i^k, is a function of Z_i because Country i's deterrence, or Z_i, prevents the war against Country i by Enemy k, or q_i^k. Thus, as Z_i increases, q_i^k decreases. More specifically, q_i^k should be between 0 and 1, and a concave-down-function of Z_i; dZ_i/dq_i^k is negative, and $d^2 Z_i/d^2 q_i^k$ is positive. Therefore, it is assumed that Country i would estimate q_i^k by an efficient factor of the deterrence for War k, or d_i^k, as follows:

$$q_i^{\,k} = \exp\left(-d_i^{\,k} Z_i\right).$$

Generally, d_i^k should also be a value that is greater than 0. If $d_i^k=0$, Z_i does not have any effect on deterrence and Country i perceives that War k will occur regardless of Z_i; if $d_i^k>0$, Z_i is effective and q_i^k decreases as Z_i increases.

Concerning the two wars, the situations in the model are classified into four cases. In Situation 0, neither War A nor War B occurs; this is peacetime. In Situation 1, only War A occurs. In Situation 2, War B occurs. In Situation 3, both War A and War B occur. Since only four situations are possible, the sum of the probabilities of war in each situation, or P_i^s, must satisfy:

$$\sum_{s=0}^{3} P_i^s = 1$$

Using q_i^k, P_i^s is denoted by the following four equations:

$$P_i^0 = 1 - q_i^A - q_i^B + q_i^A q_i^B$$

$$P_i^1 = q_i^A \left(1 - q_i^B\right)$$

$$P_i^2 = q_i^B \left(1 - q_i^A\right)$$

$$P_i^3 = q_i^A q_i^B$$

4. Utilities at Peacetime and at War

The utilities of the countries in each situation are measured by their expenditure for private goods. The expenditure for private goods in the case of a war is less than that during peacetime. The reduction in private goods is called a loss by war. Country i estimates the loss suffered under Situation s, or L_i^s, would be a portion of its GDP, or G_i. When s = 0 during peacetime, Country i loses nothing and l_i^0 is zero. Compared to Situation 0, Country i estimates the loss under Situation s at l_i^s of G_i. Then,

$$L_i^s = l_i^s G_i \quad (0 \le l_i^s \le 1).$$

For convenience, suppose that loss from both wars, or l_i^3, is the sum of the losses from each war. Thus, the following equation denotes l_i^3:

$$l_i^3 = l_i^1 + l_i^2 .$$

Including the loss, Country i estimates its utility under Situation s by the following equation, where a_i is its utility elasticity:

$$U_i^s = \left(X_i - L_i^s\right)^{a_i} .$$

Country i's expected utility is expressed as follows:

$$U_i = \sum_{s=0}^{3} P_i^s U_i^s$$

5. Constraint

Subject to its resource constraint, each country in this model selects its own defense expenditure to maximize its expected utility. The country's resource is its gross domestic product (GDP), which is denoted by G_i. Country i's constraint is represented by the equation below, where p_i is the price of the private goods relative to the defense goods.

$$G_i = p_i X_i + Y_i$$

As described above, Country 1 responds to Country 2's defense expenditure and determines its defense expenditure to maximize its expected utility subject to its resource constraint; and Country 2 does so as well. By responding to one another, defense expenditures in both countries approach an equilibrium point, called a Nash equilibrium.

B. ALLIANCE MODEL WITH RISK PERCEPTION

Whereas previous alliance models see an alliance as providing at least partially public goods, which benefit all allies, I would like to emphasize that an alliance may involve risks as well as benefits.

The following contingencies could be happen. Once a country joins an alliance, it would have to deal with all wars with which an ally of the alliance is associated.[1] Thus, the country could be dragged into a war in which it would not be involved without the alliance, and would suffer some loss from the war. In addition, once a country joins an alliance, it upsets the balance of power surrounding the alliance. This could lead to an increased probability of war involving the alliance, increasing the cost of war for all alliance members. These contingencies could decrease the utility of the country joining an alliance. We

[1] The exception is alliances that specify a common adversary. [Ref. 9]

call the case where forming an alliance reduces utility of the allies as an alliance with risk.

For this reason, there are many countries that hold a variety of perceptions regarding the risk of alliances. For example, countries that have had bad experiences with wars may perceive risk from an alliance rather than benefit. On the other hand, countries that have won wars and use the military as a political tool, may perceive mainly benefits from an alliance rather than risk. In the alliance model without risk, the alliance is perceived as pure or impure public goods. In other words, both countries always perceive a benefit from the alliance.

Suppose that there are two types of countries in terms of the benefit and the risk from an alliance: Type 1 is a country that perceives only benefits from the alliance, like the alliance model without risk. Type 2 is a country that perceives the risk from the alliance as well as the benefit.

1. Type of Country

With regards to the risk and benefit perception, consider two types of countries, Type 1 and Type 2. To create the risk alliance model, I would like to focus attention on the probability of wars and change the alliance model without risk to incorporate risk perceptions.

Type 1 countries perceive only benefits. Like the alliance model without risk, it is assumed that the country perceives that the alliance could decrease the probabilities of wars against not only itself but also the other country joining the alliance. Thus, the contribution by Type 1 Country to the other country joining the alliance could increase Type 1 Country's utility; the contribution by the other country to Type 1 Country also increases Type 1 Country's utility.

Type 2 Country perceives the risk as well as the benefit. In contrast to Type 1 Country, Type 2 Country perceives that an alliance decreases the probability of war against only itself but increases the probability that it will be drawn into a war against the other country joining the alliance. Thus, the

contribution by Type 2 Country to the other country joining the alliance can reduce Type 2 Country's utility if its ally is involved in a war, but the contribution by the other country to Type 2 Country increases Type 2 Country's utility. Simply put, Type 2 Country welcomes help from the other country, but perceives potential cost if it helps the other country.

Let us discuss the types of countries mathematically. This analysis changes the probability of the war function in the alliance model without risk perception to reflect risk perceptions. Except for the probabilities of wars, Type 1 and Type 2 are represented by the same equations regarding its objective and constraint.

a. Type 1

Type 1 Country's perception of the probabilities of war is expressed below, like the alliance model without risk perception.

$$q_i^{\,k} = \exp\left(-d_i^{\,k}\left(Y_i + E_{ij}Y_j\right)\right), \text{ for i, j = 1,2 and } i \neq j$$

According to this equation, as E_{ij} and E_{ji} increase unilaterally, probabilities of Wars k and m, q_i^k and q_j^m, will decrease respectively. In other words, the higher level of cooperation makes the probability of both wars decrease.

b. Type 2

Type 2 Country's perception about the probabilities of Wars k and m is expressed by the two equation below.

$$q_i^{\,k} = \exp\left(-d_i^{\,k}\left(Y_i + E_{ij}Y_j\right)\right)$$

$$q_i^{\,m} = \exp(-d_i^{\,m}(Y_j - E_{ji}Y_i))$$

for i, j= 1,2 and $i \neq j$

When i =1, k=A and m=B.

When i =2, k=B and m=A

The difference compared to the Type 1 country is the minus sign in the second equation above; this sign was positive in the equation for Type 1 Country. In the second equation, $(Y_j - E_{ji}Y_i)$ is used instead of $(Y_j + E_{ji}Y_i)$. As a result, q_j^m will increase if E_{ji} increases unilaterally, while q_i^k will decrease if E_{ij} increases unilaterally. Thus, the higher level of cooperation makes the probability of War k decrease. However, it makes the probability of War m increase. As a consequence, it can be presumed that risk has been introduced into the alliance model.

2. Combination of Countries

In this paper, presume three combinations of countries. Combination 1 is Type 1 and Type 1, Combination 2 is Type 2 and Type 2, and Combination 3 is Type 1 and Type 2.

a. Combination 1

Both Countries 1 and 2 are Type 1 countries. See Appendices 1 and 2. They perceive benefits from the alliance. Country 1 observes both Enemies A and B as adversaries as does Country 2. In other words, they share common adversaries like NATO did during the Cold War. At that time, the Soviet Union had been extending its domain and increasing its influence on surrounding countries. Aggression by the Soviet Union became the biggest threat to all western countries including the United States. To deter aggression, the countries established NATO where all members shared a common adversary, the Soviet Union.

b. Combination 2

Both Countries 1 and 2 are Type 2 Countries. See Appendices 3 and 4. Both Countries 1 and 2 perceive the risk and benefit of an alliance. In other words, Country 1 does not consider Enemy B, only Enemy A as its adversary, while Country 2 does not consider Enemy A, only Enemy B as its adversary.

This combination, for example, represents the current situation at NATO. After the Cold War, as the Soviet Union collapsed, the threat to western countries has been decreasing. Additionally, as the eastern countries begin to join NATO, the common adversaries are becoming more and more ambiguous; each member faces increasingly unique adversaries.

Another example is peacekeeping operations, PKO, by the United Nations. While U.N. members think that the U.N. is valuable for their security, they also think that there are risks involved in cooperating with the PKO. The wars among the countries under the current PKO do not affect all members. There is no common nor obvious adversaries for all U.N. members.

c. Combination 3

Country 1 is Type 2 and Country 2 is Type 1. See Appendices 5 and 6. Country 1 perceives both the risk and the benefit of the alliance, while Country 2 perceives mainly the benefit of the alliance. In other words, all adversaries for Country 1 are adversaries of Country 2, while only some of the adversaries of Country 2 are adversaries of Country 1. Enemy A, the main adversary of Country 1, is an adversary of Country 2; Enemy B, the main adversary of Country 2, is not the adversary of Country 1.

An example is the relationship between the United States and Japan. The United States thinks that all of the enemies that may attack Japan are its adversaries. However, Japan does not feel that all of the enemies facing the United States are adversaries of Japan. Japan plans to use its military to defend only itself and does not think that its armed forces may operate except in the Far East. [Ref. 8]

C. NASH EQUILIBRIUM

Nash equilibrium determines each country's defense expenditure. The equations representing the country's characters are shown in Appendices 1 to 6. As its reaction curve equation shows, Country 1 responds to the amount of the Country 2's defense expenditure and decides on its defense expenditure to

maximize its expected utility subject to its resource constraint. Country 2 does the same as its reaction curve equation shows. As they respond to one another, their defense expenditures approach an equilibrium point called a Nash equilibrium.

From a graphical standpoint, a reaction curve represents the amount of defense expenditure that one of the countries selects in response to another country's defense expenditure. Both countries have their own reaction curve. The point where the reaction lines cross one another is their Nash equilibrium point.

To illustrate the three combinations, suppose that there are two alliance members and that the GDP and the parameters take the values indicated in Table 1. These values depict the two countries as identical with the exception of the GDP.

	G_i	ai	di1	di2	li0	li1	li2	li3
Country 1	80	0.5	0.2	0.2	0	0.4	0.2	0.6
Country 2	100	0.5	0.2	0.2	0	0.2	0.4	0.6

Table 1. Illustrative Parameter Values

For example, look at the reaction curves, where $E_{12} = E_{21} = 0.0, 0.5, 1.0$, under Combinations 1, 2, and 3.

1. Combination 1

Figure 1 shows reaction curves in Combination 1. In the figure, dotted curves represent Country 1's reaction curves, solid curves represent the Country 2's reaction curve, and solid dots represent Nash equilibrium points. Table 2 shows each country's defense expenditures at the Nash equilibrium points. If the parameters are changed, the reaction curves and the Nash equilibrium points will move.

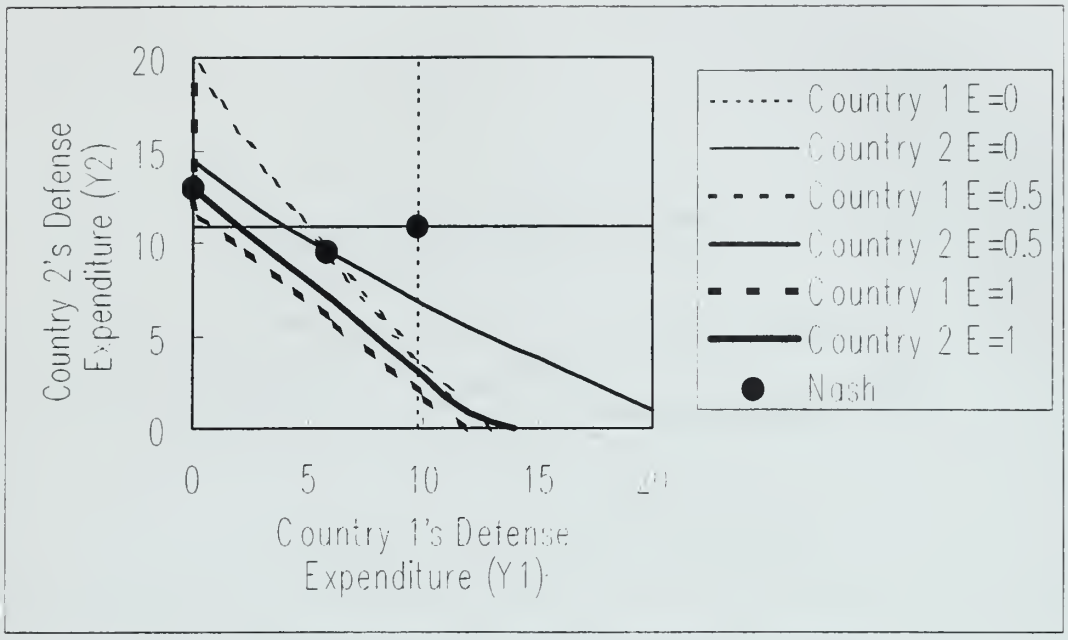

Figure 1. Reaction Curve in Combination 1

	Country 1	Country 2
$E_{12}=E_{21}=0.0$	9.76	10.92
$E_{12}=E_{21}=0.5$	5.74	9.56
$E_{12}=E_{21}=1.0$	0	12.92

Table 2. Defense Expenditure at Nash Equilibrium Point in Combination 1

2. Combination 2

Figure 2 shows reaction curves in Combination 2, and Table 3 shows defense expenditures at the Nash equilibrium points.

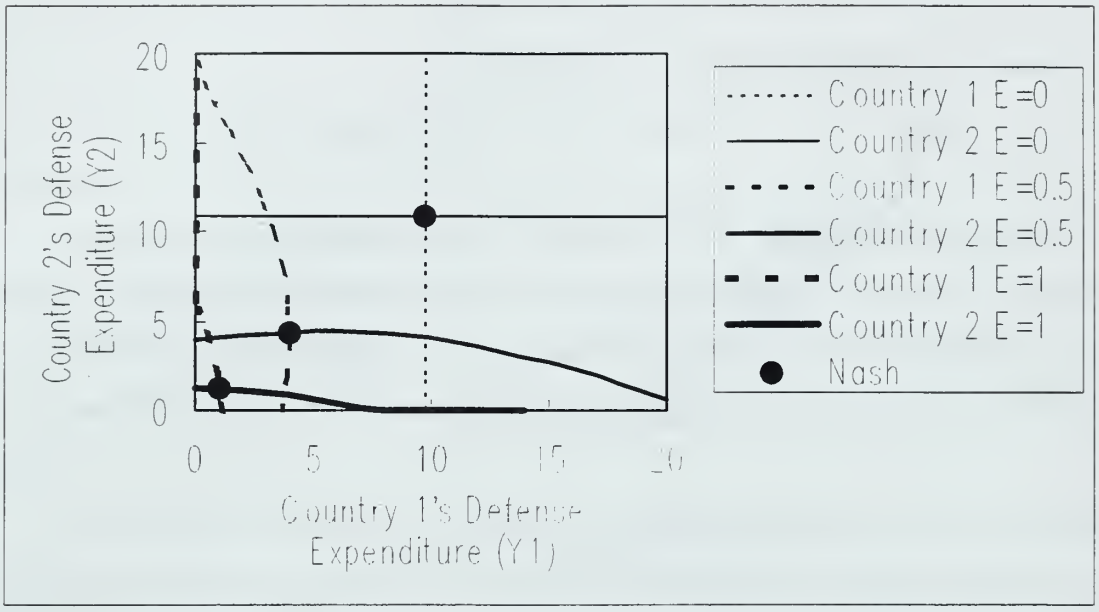

Figure 2. Reaction Curve in Combination 2

	Country 1	Country 2
$E_{12}=E_{21}=0.0$	9.76	10.92
$E_{12}=E_{21}=0.5$	3.94	4.38
$E_{12}=E_{21}=1.0$	1.00	1.20

Table 3. Defense Expenditure at Nash Equilibrium in Combination 2

3. Combination 3

Figure 3 shows reaction curves in Combination 3, and Table 4 shows defense expenditures at the Nash equilibrium points.

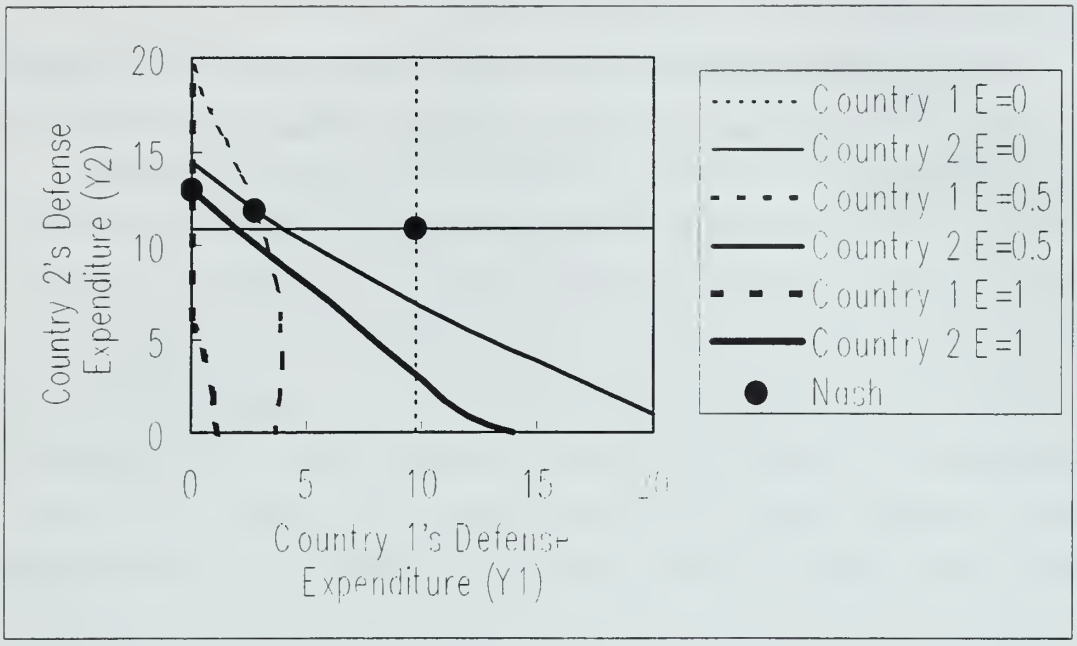

Figure 3. Reaction Curve in Combination 3

	Country 1	Country 2
$E_{12}=E_{21}=0.0$	9.76	10.92
$E_{12}=E_{21}=0.5$	2.78	11.90
$E_{12}=E_{21}=1.0$	0.00	12.92

Table 4. Defense Expenditure at Nash Equilibrium Point in Combination 3

From these examples, we can determine each country's defense expenditures from the graphical representation. In these figures, the Nash equilibrium approaches the point where their reaction lines cross one another as both countries continue to interact, regardless of the initial standing point.

While Countries 1 and 2 in Combination 1 have a similar shaped reaction curves, since they are the same type of countries, countries in Combination 3 have different shaped reaction curves. In Combination 3, Country 1 tends to reduce its defense expenditure as the level of cooperation increases because it perceives negative impact on its utility. As a result, the disproportionality of defense burden between the countries in Combination 3 is greater than that of

Combination 1 or 2.

The next chapter examines each country's utility at the Nash equilibrium point relative to the isolation case and considers whether or not forming an alliance involves risk.

III. RESULTS

Using the alliance models with risk, I would like to examine the three combinations in terms of the expected utilities at Nash equilibrium and consider the effect of an alliance on both utility and the implications for alliance formation. In Combination 1, both Countries 1 and 2 are Type 1 countries, which perceive benefits from alliance. In Combination 2, both Countries 1 and 2 are Type 2 countries, which perceive risks and benefits of alliances. In Combination 3, Country 1 is a Type 2 country and Country 2 is a Type 1 country. Country 1 perceives risks and benefits of the alliance, while Country 2 only perceives benefits.

For each combination, I will discuss both a symmetrical case and an asymmetrical case regarding E_{ij}. The symmetrical case is defined as a case where E_{12} is equal to E_{21}. The asymmetrical case is defined as the case where E_{12} may not equal E_{21}. Using the symmetrical case, the relationship between the expected utility and defense expenditure is discussed. The relationship of expected utility between Countries 1 and 2 is discussed in the asymmetrical case.

To illustrate the symmetric case, suppose that the parameters of the two countries in each combination take the values indicated in Table 5. These values imply that the two countries are identical with the exception of GDP.

	Gi	a_i	di1	di2	li0	li1	l_i^2	l_i^3
Country1	80	0.5	0.2	0.2	0	0.4	0.2	0.6
Country2	100	0.5	0.2	0.2	0	0.2	0.4	0.6

Table 5. Illustrative Parameters

A. COMBINATION 1

1. Symmetrical Case

Table 6 shows defense expenditures, expected utilities, and the difference in expected utility between the isolation and alliance case. ΔU1 and ΔU2 denote the difference in Country 1's and Country 2's expected utility between isolation and alliance, respectively.

E_{ij} (E_{12}=E_{21})	Y1	Y2	U1, ΔU1	U2, ΔU2
0	9.76	10.92	7.95, 0.00	9.00, 0.00
0.25	7.76	9.74	8.12, +0.17	9.12, +0.12
0.5	5.74	9.56	8.27, +0.33	9.17, +0.17
0.75	1.60	12.12	8.54, +0.60	9.07, +0.07
1.0	0.00	12.92	8.72, +0.77	9.06, +0.06

Table 6. Expected Utility and Defense Expenditure

Figure 4 shows the relationship between defense expenditures and expected utilities graphically.

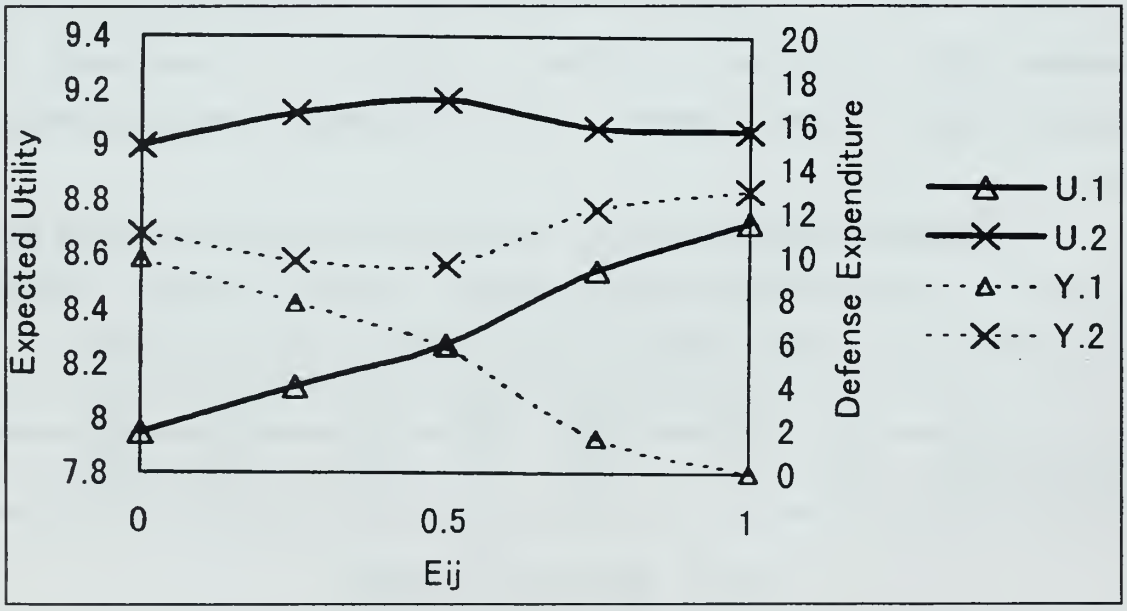

Figure 4. Expected Utility and Defense Expenditure

From the figure, we can see that Country 1's utility increases with E_{ij}, while Country 2's utility can increase or decrease depending on the value of E_{ij}.

For Country 1, the increase in E_{ij} increases the productivity of defense expenditures and decreases its burden. Therefore, the slope of the curve representing Country 1's utility is positive.

On the other hand, the slope of the curves representing Country 2's utility and defense expenditure wave. When E_{ij} is less than 0.5, the slope of the curve representing U2 can be positive; Country 2 decreases its defense expenditure as the productivity of its defense expenditure increases. In contrast, the slope is negative when E_{ij} is between 0.5 and 0.75; as Country 1 perceives a larger contribution from its larger ally, its defense expenditures decrease at an increasing rate. Country 2's defense expenditures must increase to compensate. This creates disproportional burden sharing due to an increase in E_{ij}. In this range, Country 2's defense expenditures increase, while Country 1's defense expenditure decreases in the Nash equilibrium. In this way, the net implications for Country 2 depend on the balance of the increase in productivity and the increase in alliance burden.

2. Asymmetrical Case

Figure 5 shows the relationship between the utilities of Countries 1 and 2 at various levels of cooperation using circles. The utilities of the countries in the isolation case are represented by a solid dot. Let us divide the field in the figure into four regions, based on the isolation point, and consider the region in which the circles appear.

Figure 5 shows the upper left region, named Region 1, includes cases where alliance is better than isolation, in terms of utility, for Country 2 but not for Country 1. The upper right region is Region 2, where alliance is better for both Countries 1 and 2. The lower left region is Region 3 where alliance is worse for Countries 1 and 2. The lower right region is Region 4 where alliance is better for Country 1 but worse for Country 2.

As we see in Figure 5, almost all circles are in Region 2; some circles are in Regions 1 and 4, but these points are near the boundary of Region 2. These results indicate that an alliance can benefit both countries as the level of cooperation increases. Thus, there is a strong possibility that the countries would form an alliance.

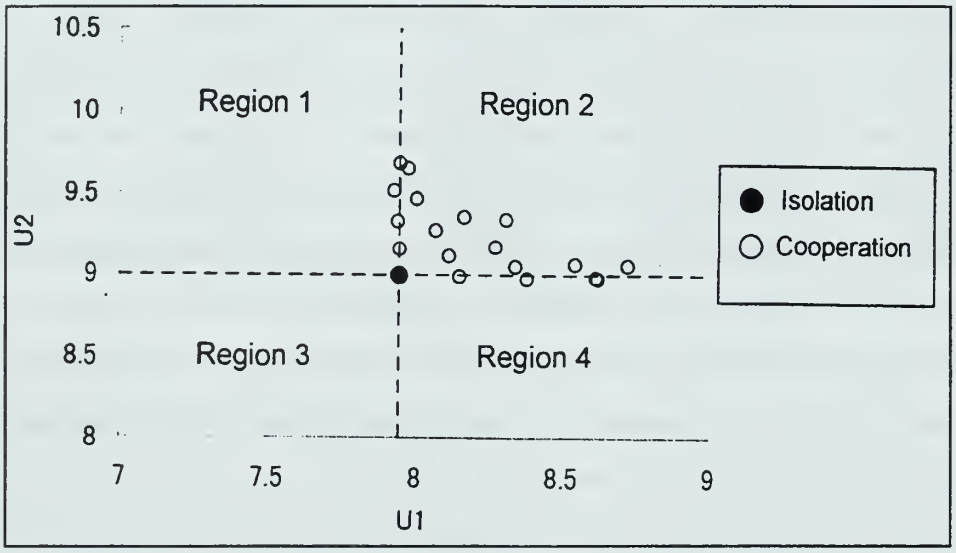

Figure 5. Utility Relationship Between Countries 1 And 2

It is necessary to look at the details of the change in these utilities to consider when the utilities in alliance are lower than that of isolation. Table 7 shows Countries 1 and 2's utilities at different levels of cooperation, E_{12} and E_{21}. In the table, the upper number in a cell represents the expected utility and the lower number denotes the difference in the expected utility between the isolation and alliance case, or the utility in alliance minus that of isolation. The light colored cells represent cases where the expected utilities of either or both countries are less than the isolation case.

	E12=0		E12=0.25		E12=0.5		E12=0.75		E12=1.0	
E21=0	U1= 7.95	U2= 9.00	U1= 8.15	U2= 8.99	U1= 8.38	U2 8.98	U1= 8.62	U2= 8.98	U1= 8.72	U2= 9.06
	ΔU1= 0.00	ΔU2= 0.00	ΔU1= 0.20	ΔU2= 0.00	ΔU1= 0.43	ΔU2= -0.02	ΔU1= 0.67	ΔU2= -0.02	ΔU1= 0.77	ΔU2= 0.06
E21=0.25	U1= 7.95	U2= 9.16	U1= 8.12	U2= 9.12	U1= 8.34	U2= 9.05	U1= 8.62	U2= 8.98	U1= 8.72	U2= 9.06
	ΔU1= 0.00	ΔU2= 0.16	ΔU1= 0.17	ΔU2= 0.12	ΔU1= 0.39	ΔU2= 0.06	ΔU1= 0.67	ΔU2= -0.02	ΔU1= 0.77	ΔU2= 0.06
E21=0.5	U1= 7.94	U2= 9.33	U1= 8.07	U2= 9.27	U1= 8.27	U2= 9.17	U1= 8.61	U2= 8.99	U1= 8.72	U2= 9.06
	ΔU1= -0.01	ΔU2= 0.33	ΔU1= 0.12	ΔU2= 0.27	ΔU1= 0.33	ΔU2= 0.17	ΔU1= 0.66	ΔU2= -0.01	ΔU1= 0.77	ΔU2= 0.66
E21=0.75	U1= 7.93	U2= 9.52	U1= 8	U2= 9.47	U1= 8.16	U2= 9.35	U1= 8.54	U2= 9.07	U1= 8.72	U2= 9.06
	ΔU1= -0.02	ΔU2= 0.52	ΔU1= 0.06	ΔU2= 0.47	ΔU1= 0.22	ΔU2= 0.35	ΔU1= 0.60	ΔU2= 0.07	ΔU1= 0.77	ΔU2= 0.06
E21=1.0	U1= 7.94	U2= 9.68	U1= 7.94	U2= 9.68	U1= 7.97	U2= 9.65	U1= 8.31	U2= 9.34	U1= 8.72	U2= 9.06
	ΔU1= 0.00	ΔU2= 0.69	ΔU1= 0.00	ΔU2= 0.69	ΔU1= 0.03	ΔU2= 0.66	ΔU1= 0.36	ΔU2= 0.34	ΔU1= 0.77	ΔU2= 0.06

Table 7. Countries 1 and 2's Utility in Combination 1

As seen in the table, Country 1's utility is 8.72 with complete cooperation, where $E_{12} = E_{21} = 1$, and 7.95 in isolation, where $E_{12} = E_{21} = 0$. If Country 1 has only these two options, there is no doubt that Country 1 would cooperate with Country 2.

But what happens if Country 1 could reduce its commitment to Country 2. In the case where $E_{12} = 1$, Country 1 does not spend any resources on defense. The contribution by Country 1 to Country 2 is always 0 regardless of E_{21}. This means that Country 1's utility is not influenced by E_{21}. In the case where E_{12} is not 1, however, Country 1 devotes resources to defense expenditures and contributes a portion of its defense expenditure to Country 2. Consider the case where $E_{12} = 0.75$. If Country 1 changes E_{21} from 1 to 0.75, 0.5, 0.25, and 0, its utility increases from 8.31 to 8.54, 8.61, and 8.62. Likewise, in the case where E_{12}=0.5, 0.25, 0.0, as Country 1 reduces E_{21} unilaterally, its utility increases.

Similarly, Country 2 also gets higher utility by reducing its commitment to Country 1. In the case where E_{21} is 1, if Country 2 changes E_{12} from 1 to 0.75, 0.5, 0.25, and 0, its utility increases from 9.06 to 9.34, 9.65, and 9.68.

This situation is like the prison dilemma. If each of the two countries can disclose its reduction in contribution to the other, both countries are likely to reduce their contribution. Ultimately, they would isolate themselves from each other. As a consequence, both countries might move toward isolation even though cooperation is clearly better for both than isolation.

If the countries do not communicate with each other and exchange information on the level of the contribution, the instability of the alliance increases, even though an alliance would benefit both countries. Hence, communication is critical in forming an alliance, even if the potential members mainly perceive a benefit from an alliance. Assuming both countries continue to exchange and monitor information about their alliance commitments, there is a higher probability that the countries in Combination 1 would form an alliance that benefits both countries.

B. COMBINATION 2

1. Symmetrical Case

Table 8 shows the defense expenditure, expected utilities, and differences in expected utility between the isolation and alliance cases. $\Delta U1$ and $\Delta U2$ denote the differences in Country 1's and Country 2's utilities, respectively.

From the table, if E_{ji} increases, the expected utilities of both countries decrease, suggesting that alliance would be worse for both countries.

Figure 6 shows the relationship between the amount of defense expenditures and the expected utilities graphically. The figure shows again that expected utilities and defense expenditures decrease if E_{ji} increases. In Combination 2, Country 1 perceives that more cooperation increases the probability of War B but decreases the probability of War A, and Country 2

E_{ij} ($E_{12}=E_{21}$)	Y1	Y2	U1, ΔU1	U2, ΔU2
0	9.76	10.92	7.95, 0.00	9.00, 0.00
0.25	6.68	7.54	7.85, -0.10	8.79, -0.21
0.5	3.94	4.38	7.44, -0.50	8.28, -0.71
0.75	2.08	2.34	6.92, -1.03	7.66, -1.33
1.0	1.00	1.20	6.46, -1.49	7.12, -1.88

Table 8. Expected Utility and Defense Expenditure

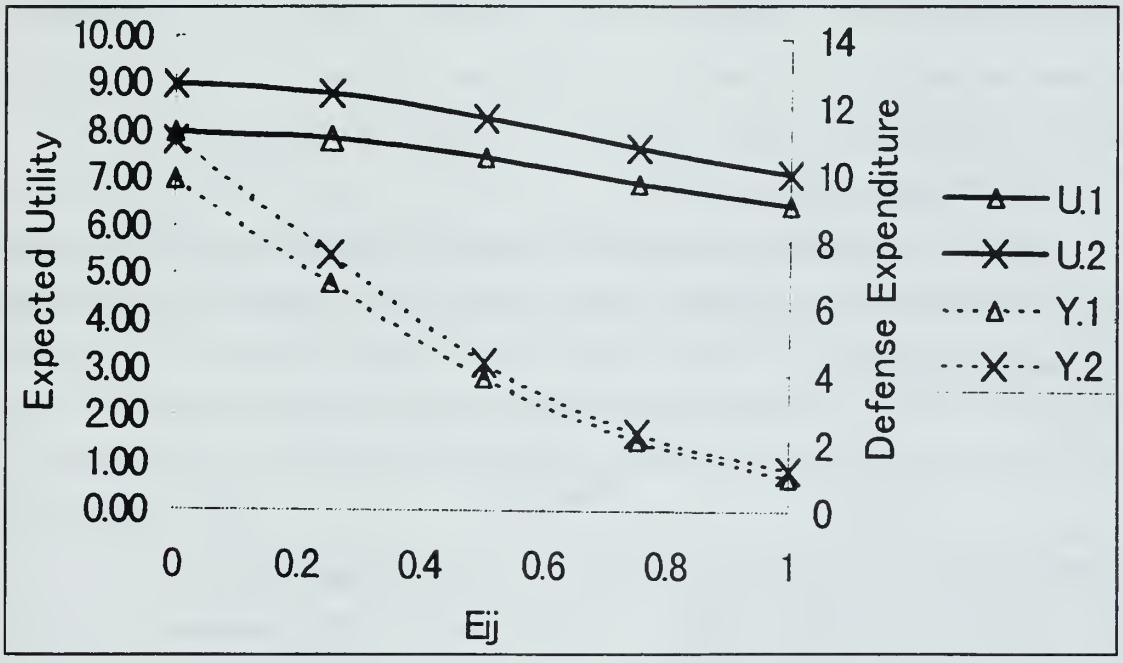

Figure 6. Expected Utility and Defense Expenditure

perceives that more cooperation increases the probability of War A but decreases the probability of War B. In this example, both countries reduce their defense expenditure as the cooperation level increases; both countries receive a larger contribution from their ally toward avoiding their own war, and their own defense expenditures have a greater adverse impact on the probability of war for their ally. Since both countries reduce defense expenditures, the alliance is not better for either country in this example. Consequently, there is a very low probability that the countries in Combination 2 will form an alliance.

However, an alliance might be better for Country 1, if the probability of War B is much lower compared to that of War A, or if the expected damage Country 1 suffers from War B is 0 or much smaller than the decrease in expected damage from War A. In this example, the same value of 0.2 is assumed for the coefficients of the probabilities, d_1^A and d_1^B as well as d_2^A and d_2^B. In addition, the damage from War A is twice as great as that of War B. The damage from War A is 40% of GDP; the damage from War B is 20% of the GDP. In this example, the effect of the alliance on the probabilities of war A and B indicate that an alliance is worse for Country 1. In the same way, an alliance is also worse for Country 2. Whether an alliance is a benefit or risk depends on the ratio of the damage from War A to War B and the impact of alliance on respective probabilities of war.

2. Asymmetrical Case

Figure 7 graphs the relationship between the utilities of Countries 1 and 2 at the various levels of cooperation using circles, and the utilities in the isolation case using a solid dot.

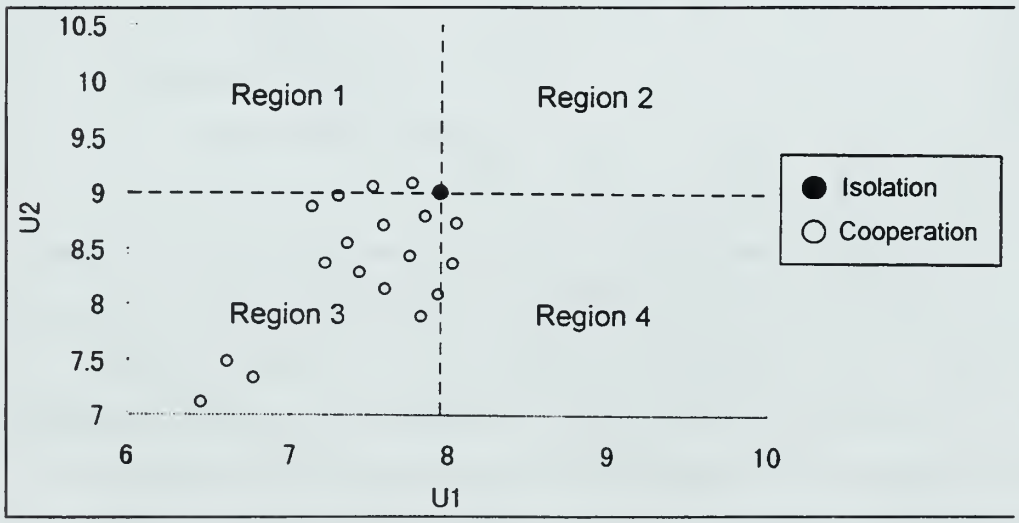

Figure 7. Utility Relationship Between Countries 1 And 2

As Figure 7 shows, the circles are almost all distributed in Region 3, where cooperation is worse for both Countries 1 and 2. Some circles are in Region 1 and Region 4 where only one of the two countries benefits from an alliance but they are near the border of Region 3. The two circles in Region 1 represent the case where $E_{12} = 0$ and $E_{21} = 0.25$ and 0.5, and the two circles in Region 4 represent the case where $E_{21} = 0$ and $E_{12} = 0.25$ and 0.5. This means that one of the countries benefits from an alliance only when the other country unilaterally contributes to its defense without expecting compensation. According to Figure 7, it is impossible for both countries to benefit from an alliance in this example.

Table 9 shows Countries 1 and 2's utilities at different levels of cooperation, E_{12} and E_{21}. In the table, the upper number in a cell represents the expected utility and the lower number denotes the difference in the expected utility between isolation and alliance, (i.e., the utility in alliance minus that of isolation). The lightly shaded cells represent cases where the expected utility of one country is higher than the isolation case, and the darkly colored cells represent the cases where the expected utilities of both countries are less than the isolation case.

This table further illustrates the difficulty in forming an alliance between these countries. It is unreasonable for either country to increase their commitment to the other country because their utilities decrease as the level of cooperation increase. Therefore, there is little possibility that the countries in Combination 2 would form an alliance.

	E12=0		E12=0.25		E12=0.5		E12=0.75		E12=1.0	
E21=0	U1=	U2=	U1=	U2=	U1=	U2=	U1=	U2=	U1=	U2=
	7.95	9	8.05	8.72	8.02	8.37	7.93	8.08	7.83	7.89
	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=
	0	0	0.1	-0.28	0.08	-0.63	-0.01	-0.91	-0.12	-1.1
E21=0.25	U1=	U2=	U1=	U2=	U1=	U2=	U1=	U2=	U1=	U2=
	7.77	9.08	7.85	8.79	7.76	8.43	7.6	8.14	7.42	7.96
	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=
	-0.18	0.07	-0.1	-0.21	-0.18	-0.57	-0.35	-0.86	-0.53	-1.03
E21=0.5	U1=	U2=	U1=	U2=	U1=	U2=	U1=	U2=	U1=	U2=
	7.52	9.06	7.59	8.71	7.44	8.28	7.21	7.96	6.96	7.78
	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=
	-0.43	0.06	-0.36	-0.28	-0.5	-0.71	-0.74	-1.04	-0.74	-1.2
E21=0.75	U1=	U2=	U1=	U2=	U1=	U2=	U1=	U2=	U1=	U2=
	7.3	8.98	7.36	8.54	7.19	8.03	6.92	7.66	6.62	7.47
	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=
	-0.65	-0.02	-0.58	-0.45	-0.76	-0.96	-1.03	-1.33	-1.33	-1.53
E21=1.0	U1=	U2=	U1=	U2=	U1=	U2=	U1=	U2=	U1=	U2=
	7.14	8.88	7.22	8.36	7.05	7.77	6.77	7.34	6.46	7.12
	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=	ΔU1=	ΔU2=
	-0.81	-0.12	-0.73	-0.63	-0.89	-1.23	-1.17	-1.66	-1.49	-1.85

Table 9. Countries 1 and 2's Utility in Combination 2

This table further illustrates the difficulty in forming an alliance between these countries. It is unreasonable for either country to increase their commitment to the other country because their utilities decrease as the level of cooperation increase. Therefore, there is little possibility that the countries in Combination 2 would form an alliance.

C. COMBINATION 3

1. Symmetrical case

Table 10 shows the defense expenditures, the expected utilities, and the differences in expected utility between the isolation and alliance cases. ΔU1 and ΔU2 denote the differences in Country 1's and Country 2's expected utility between the isolation and alliance cases, respectively. A colored cell represents the case where an alliance is worse for one of the countries. Country 1 is a smaller country of Type 2 that perceives risks as well as benefits from the alliance while Country 2 is a larger country of Type 1 that perceives benefits from the alliance.

E_{ij} ($E_{12} = E_{21}$)	Y1	Y2	U1, ΔU1	U2, ΔU2
0	9.76	10.92	7.95, 0.00	9.00, 0.00
0.25	6.44	10.30	8.04, +0.09	9.06, +0.06
0.5	2.78	11.90	8.30, +0.35	9.01, +0.01
0.75	0.00	13.64	8.62, +0.67	8.98, -0.02
1.0	0.00	12.92	8.72, +0.77	9.06, +0.06

Table 10. Expected Utility and Defense Expenditure

Figure 8 shows the relationship between the amount of defense expenditures and the expected utilities graphically.

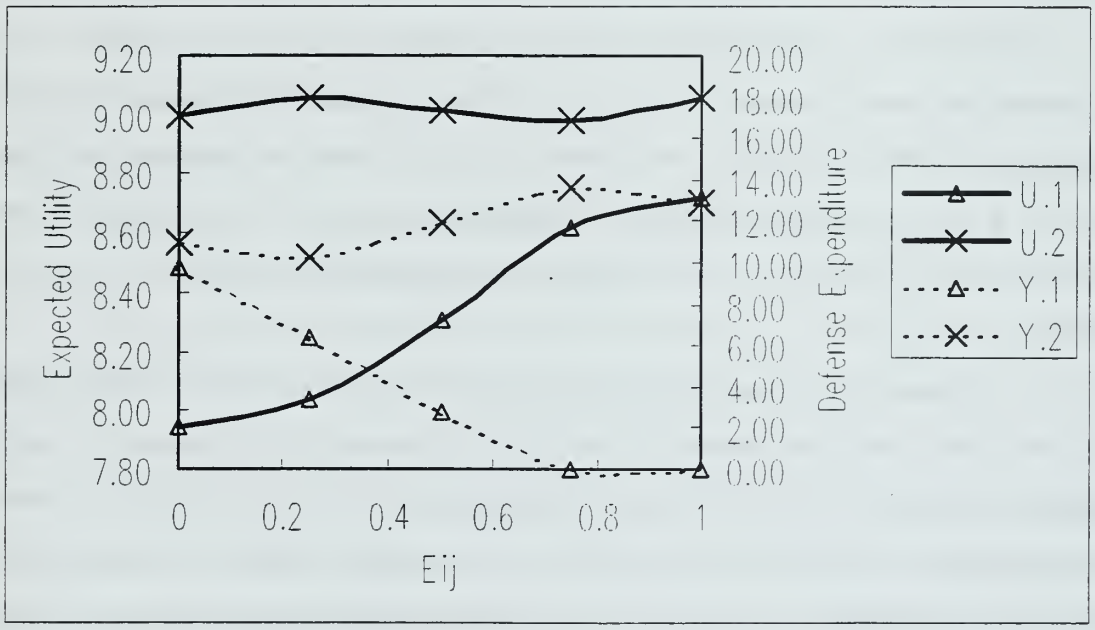

Figure 8. Expected Utility and Defense Expenditure

From the figure, we can see that Country 1's utility increases with E_{ij}, while Country 2's utility increases, decreases, and increases with E_{ij}.

Country 1 perceives risks and benefits from the alliance. As E_{ij} increases, Country 1's defense expenditures decrease the probability of War A but increases the probability of War B. This means that the alliance decreases the productivity of Country 1's defense expenditures. Because of Country 1's negative effect on the probability of War B, an increase in the level of cooperation decreases Country 1's defense expenditure more than in

Combination 1. Country 1's defense expenditures become zero when E_{ij} is 0.75 in Combination 3, while it becomes zero when E_{ij} is 1.0 in Combination 1.

On the other hand, Country 2 perceives benefits from the alliance. As E_{ij} increases, Country 2's defense expenditures decrease the probability of both Wars A and B. This means that the alliance increases the productivity of Countries 2's defense expenditure. Due to the positive effect of the alliance on the probability of War A and B, Country 2 compensates for the reduction in Country 1's defense expenditure as the level of cooperation increases.

In this way, the increase in E_{ij} decreases Country 1's burden and increases Country 2's burden. As the result, Country 1 can reduce its defense expenditure, keep the risk of the alliance low and receive contributions from Country 2 by forming an alliance. Therefore, Country 1's expected utility increases as E_{ij} increases. In the other words, Country 1 is willing to form the alliance.

In contrast, the curve representing the Country 2's expected utility rises, falls, and rises as E_{ij} increases. Country 2's expected utility increases as E_{ij} increases when E_{ij} is less than 0.25. For Country 2, the increase in E_{ij} increases the productivity of Country 2's defense expenditure, which decreases the probability of both Wars A and B. The increase in the productivity allows Country 2 to decrease its defense expenditure, and thus the slope of the curve representing Country 2's expected utility is positive in this range.

When E_{ij} is between 0.25 and 0.75, the expected utility decreases as E_{ij} increases. In this range, Country 1 reduces its defense expenditure as E_{ij} increases because of the negative impact of alliance; Country 2 increases its defense expenditure because of positive impact of the alliance. The negative effect of an increase in y_2 on Country 2's expected utility is more than the positive effect of the increase in the productivity. As the result, Country 2's expected utility decreases as E_{ij} increases.

When E_{ij} is more than 0.75, Country 2's expected utility increases as E_{ij} increases. In this range, Country 2 shoulders the entire burden since Country 1's defense expenditure is 0. Even if E_{ij} increases, the portion of the burden Country 2 shoulders does not change because Country 1 cannot reduce its defense expenditure further. As a result, the increase in the level of cooperation decreases the probability of War A and allows Country 1 to reduce its defense expenditures. Therefore, Country 2's expected utility increases as E_{ij} increases when E_{ij} is more than 0.75.

The expected utilities depend on the balance between the productivity of defense expenditures and the disproportionate level of burden sharing. The disproportionate amount of burden sharing in Combination 3 is more obvious than in Combination 1 because of Country 1's risk perception.

2. Asymmetrical case

The expected behaviors by the countries of Combination 3 in the asymmetrical case are not as simple as in Combinations 1 and 2. For this reason, let us discuss their utility after dividing Combination 3 into two cases, based on GDP. The first case is where Country 1's GDP is 10 and Country 2's GDP is 100. The second case is where Country 1's GDP is 80 and Country 2's GDP is 100. Except for differences in the GDP and risk perception, Country 1 and Country 2 are identical.

a. Case (G1 = 10, G2 = 100)

In this case, Country 1's GDP is 10 and Country 2's GDP is 100. Thus, Country 1 is much smaller than Country 2.

Figure 9 shows the relationship between the utilities of Countries 1 and 2 at various levels of cooperation using circles, and in isolation case using a solid dot. We can see that circles are located only in Region 2 where cooperation is better for both countries than isolation. Since both Countries 1 and 2 can benefit from an alliance without any loss, there is a higher probability that these two countries will form an alliance.

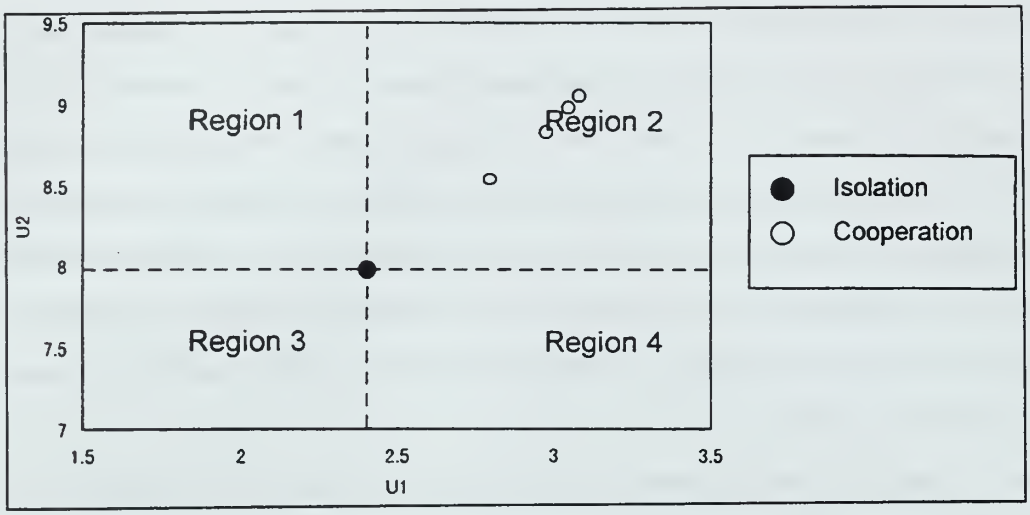

Figure 9. Utility Relationship Between Countries 1 And 2

Table 11 shows Countries 1 and 2's expected utilities in Combination 3. This further motivates the results in Figure 9 and reinforces the high probability that these countries will form an alliance.

	E12=0		E12=0.25		E12=0.5		E12=0.75		E12=1.0	
E21=0	U1= 2.40	U2= 7.98	U1= 2.79	U2= 8.54	U1= 2.97	U2= 8.83	U1= 3.05	U2= 8.98	U1= 3.08	U2= 9.06
	ΔU1= 0.00	ΔU2= 0.00	ΔU1= 0.39	ΔU2= 0.55	ΔU1= 0.57	ΔU2= 0.85	ΔU1= 0.65	ΔU2= 1.00	ΔU1= 0.68	ΔU2= 1.07
E21=0.25	U1= 2.40	U2= 7.98	U1= 2.79	U2= 8.54	U1= 2.97	U2= 8.83	U1= 3.05	U2= 8.98	U1= 3.08	U2= 9.06
	ΔU1= 0.00	ΔU2= 0.00	ΔU1= 0.39	ΔU2= 0.55	ΔU1= 0.57	ΔU2= 0.85	ΔU1= 0.65	ΔU2= 1.00	ΔU1= 0.68	ΔU2= 1.07
E21=0.5	U1= 2.40	U2= 7.98	U1= 2.79	U2= 8.54	U1= 2.97	U2= 8.83	U1= 3.05	U2= 8.98	U1= 3.08	U2= 9.06
	ΔU1= 0.00	ΔU2= 0.00	ΔU1= 0.39	ΔU2= 0.55	ΔU1= 0.57	ΔU2= 0.85	ΔU1= 0.65	ΔU2= 1.00	ΔU1= 0.68	ΔU2= 1.07
E21=0.75	U1= 2.40	U2= 7.98	U1= 2.79	U2= 8.54	U1= 2.97	U2= 8.83	U1= 3.05	U2= 8.98	U1= 3.08	U2= 9.06
	ΔU1= 0.00	ΔU2= 0.00	ΔU1= 0.39	ΔU2= 0.55	ΔU1= 0.57	ΔU2= 0.85	ΔU1= 0.65	ΔU2= 1.00	ΔU1= 0.68	ΔU2= 1.07
E21=1.0	U1= 2.40	U2= 7.98	U1= 2.79	U2= 8.54	U1= 2.97	U2= 8.83	U1= 3.05	U2= 8.98	U1= 3.08	U2= 9.06
	ΔU1= 0.00	ΔU2= 0.00	ΔU1= 0.39	ΔU2= 0.55	ΔU1= 0.57	ΔU2= 0.85	ΔU1= 0.65	ΔU2= 1.00	ΔU1= 0.68	ΔU2= 1.07

Table 11. Countries 1 and 2's Utility in Combination 3 (G1 = 10, G2 = 100)

According to the table, when E_{12} is 0.25, 0.5, 0.75, or 1.0, which includes all alliance cases, Country 1 does not spend any resources on defense. Country 2 shoulders all of the alliance burden. Country 1 is so comfortable with Country 2's contribution that Country 1 does not perceive the need to spend resources on defense. Country 1 can save its defense expenditure by forming an alliance and leaving its defense to Country 2. As the result, Country 1 benefits from the alliance. For this reason, the productivity of Country 2's defense expenditure is the only factor influencing both country's expected utility.

An alliance also benefits Country 2. According to Table 11, as E_{12} rises from 0.0 to 0.25, 0.5, 0.75, and 1.0, Country 2's utility increases from 7.98 to 8.54, 8.83, 8.98, and 9.06. From Country 2's point of view, Country 1 does not maintain sufficient deterrence. Thus, there is too a high possibility of a war against Country 1. If war occurs, Country 2 would suffer some damage. For this reason, Country 2 does not want to leave Country 1 in isolation. Country 2 is willing to form an alliance to deter War A, even though it shoulders the entire alliance burden.

Consequently, the two countries in Combination 3 are very likely to form an alliance when the Type 2 country's GDP is much smaller than the GNP of the other country.

b. Case (G1=80, G2=100)

In this case, Country 1's GDP is 80 and Country 2's GDP is 100. Country 1 perceives both risks and benefits from an alliance, and it is a little smaller than Country 2 which perceives only benefits from an alliance.

Figure 10 shows the relationship between the utilities of Countries 1 and 2 in alliance using circles, and their utilities in isolation using a solid dot. From the figure, we can see that the circles are distributed in all four regions. It is not simple to determine who receives benefits. This case is more unpredictable than the first case (G1=10, G2=100). Yet, an alliance still may be formed because it is possible that both countries can benefit from an alliance.

To see the details, Figure 10 shows the utilities of Countries 1 and 2 for different levels of cooperation.

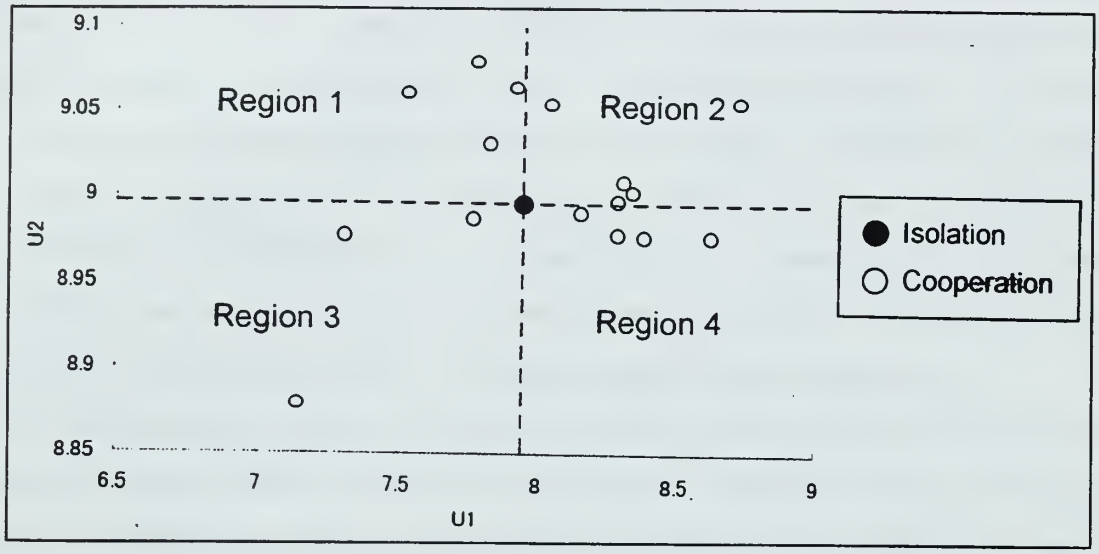

Figure 10. Utility Relationship Between Countries 1 and 2

	E12=0		E12=0.25		E12=0.5		E12=0.75		E12=1.0	
E21=0	U1= 7.95	U2= 9	U1= 8.15	U2= 8.99	U1= 8.38	U2= 8.98	U1= 8.62	U2= 8.98	U1= 8.72	U2= 9.06
	ΔU1= 0	ΔU2= 0	ΔU1= 0.2	ΔU2= 0	ΔU1= 0.43	ΔU2= -0.02	ΔU1= 0.67	ΔU2= -0.02	ΔU1= 0.77	ΔU2= 0.06
E21=0.25	U1= 7.77	U2= 9.08	U1= 8.04	U2= 9.06	U1= 8.34	U2= 9	U1= 8.62	U2= 8.98	U1= 8.72	U2= 9.06
	ΔU1= -0.18	ΔU2= 0.08	ΔU1= 0.09	ΔU2= 0.06	ΔU1= 0.39	ΔU2= 0.01	ΔU1= 0.67	ΔU2= -0.02	ΔU1= 0.77	ΔU2= 0.06
E21=0.5	U1= 7.52	U2= 9.06	U1= 7.91	U2= 9.06	U1= 8.3	U2= 9.01	U1= 8.62	U2= 8.98	U1= 8.72	U2= 9.06
	ΔU1= -0.43	ΔU2= 0.06	ΔU1= -0.03	ΔU2= 0.07	ΔU1= 0.35	ΔU2= 0.01	ΔU1= 0.67	ΔU2= -0.02	ΔU1= 0.77	ΔU2= 0.06
E21=0.75	U1= 7.3	U2= 8.98	U1= 7.82	U2= 9.03	U1= 8.29	U2= 9	U1= 8.62	U2= 8.98	U1= 8.72	U2= 9.06
	ΔU1= -0.65	ΔU2= -0.02	ΔU1= -0.13	ΔU2= 0.04	ΔU1= 0.34	ΔU2= 0	ΔU1= 0.67	ΔU2= -0.02	ΔU1= 0.77	ΔU2= 0.06
E21=1.0	U1= 7.14	U2= 8.89	U1= 7.76	U2= 8.99	U1= 8.29	U2= 8.98	U1= 8.62	U2= 8.98	U1= 8.72	U2= 9.06
	ΔU1= -0.81	ΔU2= -0.12	ΔU1= -0.19	ΔU2= -0.01	ΔU1= 0.34	ΔU2= -0.02	ΔU1= 0.67	ΔU2= -0.02	ΔU1= 0.77	ΔU2= 0.06

Table 12. Countries 1 and 2's Utility in Combination 2 (G1 = 80, G2 = 100)

According to the table, Country 1 achieves the same utility regardless of its commitment to Country 2 when E_{12} is 1 or 0.75. When E_{12} is 1

or 0.75, y_1 falls to zero so Country 1's commitment to Country 2 is irrelevant. When E_{12} is less than 0.75, Country 1 can increase its utility if it reduces E_{21} unilaterally. When E_{12} is 0.25, for example, Country 1's utility rises from 7.76 to 7.82, 7.91, 8.04, and 8.15, as it reduces E_{21} from 1 to 0.75, 0.5, 0.25, 0.0. Therefore, Country 1 has an incentive to reduce its commitment to Country 2.

On the other hand, Country 2's behavior is classified into two cases. In the first case, where E_{12} is more than 0.75, an increase in E_{12} is better for Country 2. In this case, Country 2 shoulders the entire defense burden. Since Country 1's defense expenditure is zero, the increase in E_{12} does not influence y_1. Therefore, Country 2 gains by increasing the contribution of its defense expenditure, which reduces the probability of War A.

In the second case, where E_{12} is between 0 and 0.75, Country 2's expected utility is maximized when E_{12} is between 0 and 0.5. As Country 2's commitment increases, Country 1 is likely to reduce its defense expenditures, which has a negative impact on the probability of War B. In response, Country 2 is likely to increase its defense expenditure which has a positive impact on the probability of both Wars A and B. As E_{21} increases, Country 1 gains by reducing y_1. Country 2 loses by increasing y_2 to compensate for Country 1's reduction.

As E_{12} increases, Country 1 decreases y_1 due to Country 2's increased commitment to Country 1's defense. Country 2 loses by increasing y_2 to compensate for Country 1's reduction, but it gains because of the increase in the productivity of Country 2's defense expenditure. In this case, the balance between commitment and the disproportionate burden sharing determines the E_{ij} at which utility is maximized. Since the E12 maximizing Country 2's expected utility depends on the value of E21, Country 1's behavior influences Country 2's behavior. Country 2's behavior is changeable. Thus, it is difficult to predict how Country 2 will behave. Therefore, an alliance is less stable than in the first case (G1 = 10, G2 = 100).

The possibility that the two countries in Combination 3 would form an alliance is influenced by the size of the countries. In the case where the Type 1 country, which perceives only benefits from an alliance, is much bigger than the Type 2 country which perceives the risk and the benefit of alliance, there is a high possibility of forming an alliance. The Type 1 country is afraid of an attack on the Type 2 country because of insufficient deterrence by the Type 2 country; the Type 1 country would want to intervene with the Type 2 country in order to avoid damages from a war. Even though the Type 1 country shoulders the entire burden of deterrence for the Type 2 country, the Type 1 country is willing to form an alliance to deter a war. As a result, the Type 2 country does not have to contribute to an alliance. It does not need to consider the risk from an alliance. The Type 2 country can also benefit from an alliance. Since both countries benefit, an alliance is highly probable and stable.

As the GDP in the Type 2 country increases, it starts to increase its defense expenditures and contribute to the alliance. It will not be satisfied with only the contribution of the Type 1 country. Counting on the contribution from the Type 2 country, the Type 1 country will reduce its defense contribution. This reduces the benefit of the alliance for both countries. Therefore, an alliance would be less stable and the probability of forming an alliance would be lower as the Type 2 country grows relative to the Type 1 country.

This paper discussed the case where the Type 1 country is bigger than the Type 2 country; this can be viewed as reflecting the situation where the Type 1 country is the United States and the Type 2 country is another smaller country, such as Japan. However, assuming the GDP in the Type 1 country becomes much smaller than that of the Type 2 country, the probability of forming an alliance would become much lower.

IV. CONCLUSION

This paper has introduced an alliance model with risk involving probabilities of wars. Based on the model, there is a higher probability that the countries of Combination 1 (when both countries perceive only benefits from an alliance) would form an alliance, keep it, and benefit from it, assuming they continue to exchange and monitor information about commitment to the alliance. However, it is possible that one of the countries may be negatively affected by an alliance.

Compared to Combination 1, it is more difficult for both countries to benefit from an alliance in Combination 2 (when both countries perceive both risks and benefits from an alliance). Whereas both countries may benefit from an alliance in certain specific situations, there is a much lower possibility that the countries in Combination 2 would form an alliance.

The countries of Combination 3, where the smaller country perceives both risks and benefits of an alliance, are likely to form an alliance. As the smaller country grows relative to its ally, the implications for the expected level of cooperation differs depending on the initial level of cooperation. It could turn out that an alliance is worse for both countries. Therefore, an alliance involving the countries in Combination 3 becomes less stable as the smaller country becomes bigger.

In all three combinations, the probability of wars A and B, the level of cooperation, the amount of loss from wars, countries' GDP and the countries' threat perception influence burden sharing and the productivity of their defense expenditure (i.e., level of commitment). The alliance formation varies with the delicate balance between burden sharing and productivity

APPENDIX A. THE COUNTRY 1'S CHARACTER IN COMBINATION 1

Combination 1	
Country 1	
Type	Type 1
Objective	$U_1 = (1-q_1^A-q_1^B+q_1^Aq_1^B)U_1^0+q_1^A(1-q_1^B)U_1^1+q_1^B(1-q_1^A)U_1^2 - q_1^Aq_1^BU_1^3$
Constraint	$G_1 = p_1X_1+Y_1$
Variables	X_1, Y_1
Reaction Curve	$\frac{dU_1}{dY_1} = d_1^Aq_1^A(U_1^0-U_1^1)+E_{21}d_1^Bq_1^B(U_1^0-U_1^2) - (d_1^A+E_{21}d_1^B)q_1^Aq_1^B(U_1^0-U_1^1-U_1^2+U_1^3) - \frac{(1-q_1^A-q_1^B+q_1^Aq_1^B)a_1U_1^0}{G_1-Y_1-p_1l_1^0G_1} - \frac{q_1^A(1-q_1^B)a_1U_1^1}{G_1-Y_1-p_1l_1^1G_1} - \frac{q_1^B(1-q_1^A)a_1U_1^2}{G_1-Y_1-p_1l_1^2G_1} - \frac{q_1^Aq_1^Ba_1U_1^3}{G_1-Y_1-p_1l_1^3G_1} = 0$
Probability of War A	$q_1^A = e^{-d_1^A(Y_1+E_{12}Y_2)}$
Probability of War B	$q_1^B = e^{-d_1^B(Y_2+E_{21}Y_1)}$
Utility under Situation 0	$U_1^0 = X_1^{a_1} = \left(\frac{G_1-Y_1}{p_1}\right)^{a_1}$
Utility under Situation 1	$U_1^1 = (X_1-L_1^1)^{a_1} = \left(\frac{G_1-Y_1}{p_1}-l_1^1G_1\right)^{a_1}$
Utility under Situation 2	$U_1^2 = (X_1-L_1^2)^{a_1} = \left(\frac{G_1-Y_1}{p_1}-l_1^2G_1\right)^{a_1}$
Utility under Situation 3	$U_1^3 = (X_1-L_1^3)^{a_1} = \left(\frac{G_1-Y_1}{p_1}-l_1^3G_1\right)^{a_1}$

APPENDIX B. THE COUNTRY 2'S CHARACTER IN COMBINATION 1

Combination 1	
Country 2	
Type	Type 1
Objective	$U_2 = \left(1 - q_2^A - q_2^B + q_2^A q_2^B\right) U_2^0 + q_2^A (1 - q_2^B) U_2^1 + q_2^B (1 - q_2^A) U_2^2 - q_2^A q_2^B U_2^3$
Constraint	$G_2 = p_2 X_2 + Y_2$
Variables	X_2, Y_2
Reaction Curve	$\frac{dU_2}{dY_2} = E_{12} d_2^A q_2^A (U_2^0 - U_2^1) + d_2^B q_2^B (U_2^0 - U_2^2) - (E_{12} d_2^A + d_2^B) q_2^A q_2^B (U_2^0 - U_2^1 - U_2^2 + U_2^3) - \frac{(1 - q_2^A - q_2^B + q_2^A q_2^B) a_2 U_2^0}{G_2 - Y_2 - p_2 l_2^0 G_2} - \frac{q_2^A (1 - q_2^B) a_2 U_2^1}{G_2 - Y_2 - p_2 l_2^1 G_2} - \frac{q_2^B (1 - q_2^A) a_2 U_2^2}{G_2 - Y_2 - p_2 l_2^2 G_2} - \frac{q_2^A q_2^B a_2 U_2^3}{G_2 - Y_2 - p_2 l_2^3 G_2} = 0$
Probability of War A	$q_2^A = e^{-d_2^A (Y_1 + E_{12} Y_2)}$
Probability of War B	$q_2^B = e^{-d_2^B (Y_2 + E_{21} Y_1)}$
Utility under Situation 0	$U_2^0 = X_2^{a_2} = \left(\frac{G_2 - Y_2}{p_2}\right)^{a_2}$
Utility under Situation 1	$U_2^1 = (X_2 - L_2^1)^{a_2} = \left(\frac{G_2 - Y_2}{p_2} - l_2^1 G_2\right)^{a_2}$
Utility under Situation 2	$U_2^2 = (X_2 - L_2^2)^{a_2} = \left(\frac{G_2 - Y_2}{p_2} - l_2^2 G_2\right)^{a_2}$
Utility under Situation 3	$U_2^3 = (X_2 - L_2^3)^{a_2} = \left(\frac{G_2 - Y_2}{p_2} - l_2^3 G_2\right)^{a_2}$

APPENDIX C. THE COUNTRY 1'S CHARACTER IN COMBINATION 2

Combination 2	
Country 1	
Type	Type 2
Objective	$U_1 = \left(1 - q_1^A - q_1^B + q_1^A q_1^B\right) U_1^0 + q_1^A (1 - q_1^B) U_1^1 + q_1^B (1 - q_1^A) U_1^2 - q_1^A q_1^B U_1^3$
Constraint	$G_1 = p_1 X_1 + Y_1$
Variables	X_1, Y_1
Reaction Curve	$\frac{dU_1}{dY_1} = d_1^A q_1^A (U_1^0 - U_1^1) - E_{21} d_1^B q_1^B (U_1^0 - U_1^2) - (d_1^A - E_{21} d_1^B) q_1^A q_1^B (U_1^0 - U_1^1 - U_1^2 + U_1^3) - \frac{(1 - q_1^A - q_1^B + q_1^A q_1^B) a_1 U_1^0}{G_1 - Y_1 - p_1 l_1^0 G_1} - \frac{q_1^A (1 - q_1^B) a_1 U_1^1}{G_1 - Y_1 - p_1 l_1^1 G_1} - \frac{q_1^B (1 - q_1^A) a_1 U_1^2}{G_1 - Y_1 - p_1 l_1^2 G_1} - \frac{q_1^A q_1^B a_1 U_1^3}{G_1 - Y_1 - p_1 l_1^3 G_1} = 0$
Probability of War A	$q_1^A = e^{-d_1^A (Y_1 + E_{12} Y_2)}$
Probability of War B	$q_1^B = e^{-d_1^B (Y_2 - E_{21} Y_1)}$
Utility under Situation 0	$U_1^0 = X_1^{a_1} = \left(\frac{G_1 - Y_1}{p_1}\right)^{a_1}$
Utility under Situation 1	$U_1^1 = (X_1 - L_1^1)^{a_1} = \left(\frac{G_1 - Y_1}{p_1} - l_1^1 G_1\right)^{a_1}$
Utility under Situation 2	$U_1^2 = (X_1 - L_1^2)^{a_1} = \left(\frac{G_1 - Y_1}{p_1} - l_1^2 G_1\right)^{a_1}$
Utility under Situation 3	$U_1^3 = (X_1 - L_1^3)^{a_1} = \left(\frac{G_1 - Y_1}{p_1} - l_1^3 G_1\right)^{a_1}$

APPENDIX D. THE COUNTRY 2'S CHARACTER IN COMBINATION 2

Combination 1	
Country 2	
Type	Type 2
Objective	$U_2 = \left(1 - q_2^A - q_2^B + q_2^A q_2^B\right) U_2^0 + q_2^A (1 - q_2^B) U_2^1 + q_2^B (1 - q_2^A) U_2^2 - q_2^A q_2^B U_2^3$
Constraint	$G_2 = p_2 X_2 + Y_2$
Variables	X_2, Y_2
Reaction Curve	$\frac{dU_2}{dY_2} = -E_{12} d_2^A q_2^A (U_2^0 - U_2^1) + d_2^B q_2^B (U_2^0 - U_2^2) + (E_{12} d_2^A - d_2^B) q_2^A q_2^B (U_2^0 - U_2^1 - U_2^2 + U_2^3) - \frac{(1 - q_2^A - q_2^B + q_2^A q_2^B) a_2 U_2^0}{G_2 - Y_2 - p_2 l_2^0 G_2} - \frac{q_2^A (1 - q_2^B) a_2 U_2^1}{G_2 - Y_2 - p_2 l_2^1 G_2} - \frac{q_2^B (1 - q_2^A) a_2 U_2^2}{G_2 - Y_2 - p_2 l_2^2 G_2} - \frac{q_2^A q_2^B a_2 U_2^3}{G_2 - Y_2 - p_2 l_2^3 G_2} = 0$
Probability of War A	$q_2^A = e^{-d_2^A (Y_1 - E_{12} Y_2)}$
Probability of War B	$q_2^B = e^{-d_2^B (Y_2 + E_{21} Y_1)}$
Utility under Situation 0	$U_2^0 = {X_2}^{a_2} = \left(\frac{G_2 - Y_2}{p_2}\right)^{a_2}$
Utility under Situation 1	$U_2^1 = (X_2 - L_2^1)^{a_2} = \left(\frac{G_2 - Y_2}{p_2} - l_2^1 G_2\right)^{a_2}$
Utility under Situation 2	$U_2^2 = (X_2 - L_2^2)^{a_2} = \left(\frac{G_2 - Y_2}{p_2} - l_2^2 G_2\right)^{a_2}$
Utility under Situation 3	$U_2^3 = (X_2 - L_2^3)^{a_2} = \left(\frac{G_2 - Y_2}{p_2} - l_2^3 G_2\right)^{a_2}$

APPENDIX E. THE COUNTRY 1'S CHARACTER IN COMBINATION 3

Combination 3	
Country 1	
Type	Type 2
Objective	$U_1 = \left(1 - q_1^A - q_1^B + q_1^A q_1^B\right) U_1^0 + q_1^A (1 - q_1^B) U_1^1 + q_1^B (1 - q_1^A) U_1^2 - q_1^A q_1^B U_1^3$
Constraint	$G_1 = p_1 X_1 + Y_1$
Variables	X_1, Y_1
Reaction Curve	$\frac{dU_1}{dY_1} = d_1^A q_1^A (U_1^0 - U_1^1) - E_{21} d_1^B q_1^B (U_1^0 - U_1^2) - (d_1^A - E_{21} d_1^B) q_1^A q_1^B (U_1^0 - U_1^1 - U_1^2 + U_1^3) - \frac{(1 - q_1^A - q_1^B + q_1^A q_1^B) a_1 U_1^0}{G_1 - Y_1 - p_1 l_1^0 G_1} - \frac{q_1^A (1 - q_1^B) a_1 U_1^1}{G_1 - Y_1 - p_1 l_1^1 G_1} - \frac{q_1^B (1 - q_1^A) a_1 U_1^2}{G_1 - Y_1 - p_1 l_1^2 G_1} - \frac{q_1^A q_1^B a_1 U_1^3}{G_1 - Y_1 - p_1 l_1^3 G_1} = 0$
Probability of War A	$q_1^A = e^{-d_1^A (Y_1 + E_{12} Y_2)}$
Probability of War B	$q_1^B = e^{-d_1^B (Y_2 - E_{21} Y_1)}$
Utility under Situation 0	$U_1^0 = X_1^{a_1} = \left(\frac{G_1 - Y_1}{p_1}\right)^{a_1}$
Utility under Situation 1	$U_1^1 = (X_1 - L_1^1)^{a_1} = \left(\frac{G_1 - Y_1}{p_1} - l_1^1 G_1\right)^{a_1}$
Utility under Situation 2	$U_1^2 = (X_1 - L_1^2)^{a_1} = \left(\frac{G_1 - Y_1}{p_1} - l_1^2 G_1\right)^{a_1}$
Utility under Situation 3	$U_1^3 = (X_1 - L_1^3)^{a_1} = \left(\frac{G_1 - Y_1}{p_1} - l_1^3 G_1\right)^{a_1}$

APPENDIX F. THE COUNTRY 2'S CHARACTER IN COMBINATION 3

Combination 3	
Country 2	
Type	Type 1
Objective	$U_2 = \left(1 - q_2^A - q_2^B + q_2^A q_2^B\right) U_2^0 + q_2^A (1 - q_2^B) U_2^1 + q_2^B (1 - q_2^A) U_2^2 - q_2^A q_2^B U_2^3$
Constraint	$G_2 = p_2 X_2 + Y_2$
Variables	X_2, Y_2
Reaction Curve	$\frac{dU_2}{dY_2} = E_{12} d_2^A q_2^A (U_2^0 - U_2^1) + d_2^B q_2^B (U_2^0 - U_2^2) - (E_{12} d_2^A + d_2^B) q_2^A q_2^B (U_2^0 - U_2^1 - U_2^2 + U_2^3) - \frac{(1 - q_2^A - q_2^B + q_2^A q_2^B) a_2 U_2^0}{G_2 - Y_2 - p_2 l_2^0 G_2} - \frac{q_2^A (1 - q_2^B) a_2 U_2^1}{G_2 - Y_2 - p_2 l_2^1 G_2} - \frac{q_2^B (1 - q_2^A) a_2 U_2^2}{G_2 - Y_2 - p_2 l_2^2 G_2} - \frac{q_2^A q_2^B a_2 U_2^3}{G_2 - Y_2 - p_2 l_2^3 G_2} = 0$
Probability of War A	$q_2^A = e^{-d_2^A (Y_1 + E_{12} Y_2)}$
Probability of War B	$q_2^B = e^{-d_2^B (Y_2 + E_{21} Y_1)}$
Utility under Situation 0	$U_2^0 = {X_2}^{a_2} = \left(\frac{G_2 - Y_2}{p_2}\right)^{a_2}$
Utility under Situation 1	$U_2^1 = (X_2 - L_2^1)^{a_2} = \left(\frac{G_2 - Y_2}{p_2} - l_2^1 G_2\right)^{a_2}$
Utility under Situation 2	$U_2^2 = (X_2 - L_2^2)^{a_2} = \left(\frac{G_2 - Y_2}{p_2} - l_2^2 G_2\right)^{a_2}$
Utility under Situation 3	$U_2^3 = (X_2 - L_2^3)^{a_2} = \left(\frac{G_2 - Y_2}{p_2} - l_2^3 G_2\right)^{a_2}$

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